LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – NOVEMBER 2007

ST 3810 - STATISTICAL COMPUTING - II

BB 5

Date : 29/10/2007 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

ANSWER ANY *THREE* QUESTIONS

1. The vocabulary "richness" of a text can be quantitatively described by counting the words used once; the words used twice and so forth. Based on these counts, a linguist proposed the following distances between chapters of the Old Testament book Lamentations:

Lamentations chapter

Lamentations chapter	1	2	3	4	5
1	0				
2	0.76	0			
3	2.97	0.8	0		
4	4.88	4.17	0.21	0	
5	3.86	1.92	1.51	0.51	0

Cluster the chapters of Lamentations using the Single and Complete linkage Hierarchical methods. Draw the dendrograms and compare the results.

2. Suppose the random variables X_1, X_2 and X_3 have the covariance matrix

$$\Sigma = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

Extract all principal components and their corresponding variances.

3. a) Perspiration data from 20 healthy females was analyzed. Three components, X_1 = sweat rate, X_2 = sodium content, and X_3 = potassium content, were measured and the results are as follows:

Sample mean = (4.64, 45.4, 9.965)' and $S = \begin{pmatrix} 2.879 & 10.01 & -1.81 \\ 10.01 & 199.788 & -5.64 \\ -1.81 & -5.64 & 3.628 \end{pmatrix}$

Test $H_0 : \mu = (4, 50, 10)'$ against $H_1 : \mu \neq (4, 50, 10)'$ at 10% level of significance. (17)

b) In a certain genetical experiment, the following frequencies were obtained:

AB	Ab	aB	ab
140	22	28	10

If the theory predicts the probabilities to be in the ratio

$(2+\theta)/4$, $(1-\theta)/4$, $(1-\theta)/4$, $\theta/4$,

obtain the MLE of θ and hence test the goodness of fit.

(17)

4. Let $\{X_n, n = 0,1,2,3,\ldots\}$ be a Markov chain with state space $\{0,1,2,3,\ldots\}$ and transition function p_{xy}, where $p_{01} = 1$ and for x = 1,2,3,...

$$p_{xy} = \begin{cases} p & \text{if } y = (x+1). \\ (1-p) & \text{if } y = 0, \quad 0 < p < 1. \end{cases}$$

a. Find $f_{00}^{(n)}$, for n= 1,2,3,…..

b. Find mean recurrence time of state 0.

c. Show that the chain is irreducible. Is it ergodic?

d. Find $\lim_{n\to\infty} p_{x0}^{(n)}$ for x = 0,1,2,…., whenever it exists.

e. Find the stationary distribution, if it exists.

5. a) An infinite Markov chain on the set of non-negative integers has the transition function as follows:

$p_{k0} = (k+1)/2$ and $p_{k,k+1} = 1/(k+2)$.

i) Find whether the chain is positive recurrent, null recurrent or transient.
ii) Find the stationary distribution in case it exists. (17)

b) Consider a Branching process $\{X_n, n = 0,1,2,\ldots\}$ with the initial population size $X_0 = 1$ and the following off-spring distribution:

$p_0 = 1/8$, $p_1 = 1/2$, $p_2 = 1/4$, $p_4 = 1/8$.

i) Find the mean of the population size of the n^{th} generation.
ii) What is the probability of extinction? (17)